Simple exercises

Equalities and inequalities:

Decide for which $x \in \mathbb{R}$ the following holds true:

(1) $\frac{x-2}{2x-8} \geq 1$,

(2) $\frac{x-3}{x+5} \leq \frac{2x+3}{x-4}$,

(3) $\frac{x+4}{x-2} \geq \frac{x+1}{x+3}$,

(4) $\frac{|x|-2}{x+3} \geq \frac{x+1}{x-2}$,

(5) $\frac{|x+1|}{|x-2|-3} \geq \frac{1}{x-1}$,

(6) $||x-1|-2x| = 3$,

(7) $|x+1| + |2x-3| = |2x-1| + 4$,

(8) $||x+3|-2| \geq 1$,

(9) $|x+2| - |2x-2| \geq -8$,

(10) $|x^2 + 3|x+1| - 1| = 2$,

(11) $ax^2 + 2x + 2 = a$, $a \in \mathbb{R}$,

(12) $1 \leq |ax+1| < 2$, $a \in \mathbb{R}$,

(13) $|x|x - 3ax + 5 > 0$, $a \in \mathbb{R}$,

(14) $2^{4x+1} + 3 \cdot 2^{2x+1} - 8 = 0$

(15) $3^{1+x} + 3^{1-x} < 10$,

(16) $3\left(\frac{1}{9}\right)^{t^2} = \frac{1}{9}(27)^t$,

(17) $4^x - 3 \cdot 2^{x+1} + 8 \geq 0$,

(18) $\frac{\left(\frac{1}{3}\right)^{x^2-1}}{\left(\frac{1}{9}\right)^{|x|}} > 3$,

(19) $\left(\frac{1}{2}\right)^{x^2} < 4 \cdot 8^x$,

(20) $|2^{x+2} + a| \in < -1, 3 >$, $a \in \mathbb{R}$,

(21) $\frac{2}{\sqrt{x^2-x-6}} \leq \sqrt{\frac{4}{-x^2+3x+40}}$,

(22) $\log\left(x^2 - 3x - 4\right) \geq \log\left(-x^2 + 2x + 15\right)$,

(23) $\log_{\frac{1}{3}}\left(x^2 - 3x + 3\right) \geq 0$,

(24) $\log_2^2(x) = \log_2\left(\frac{1}{16}\right) - \log_2\left(x^5\right)$,

(25) $2\log_{\frac{1}{2}}(|x|-1) \leq \log_{\frac{1}{2}}\left(x^2 - 7x + 12\right)$,

(26) $\log_{\frac{1}{2}}\left(x^2 + 3x + 2\right) \geq -3$,

(27) $\log\left(2 - |5 - |2-x||\right) \leq 0$,

(28) $\log_2\left(x^2 + c - 3\right) \in < 1, 3)$,

(29) $\sin(2x) = \sin(x)$,

(30) $2\sin(x) + \cos(x) = 1$,

(31) $2 - \cos(2x) - 3\sin(x) < 0$,

(32) $\arccos\left(-x^2 + 4x - 1\right) > \frac{\pi}{3}$,

(33) $\cos^2(x) + 2\sin(x) < a + 2$, $a \in \mathbb{R}$,

(34) $2\sin^2(|x|-1) + 3\cos(|x|-1) = 0$,

(35) $\sin^2(x) + 2\sin(x) - \cos^2(x) < 0$,

(36) $1 - |\sin(x)| = \cos^2(x)$,

(37) $\log_{\frac{1}{2}}(1 + \sin(x)) > -1$,

(38) $\mathrm{tg}(5x-1) \leq \sqrt{3}$,

(39) $\left(\frac{\left(\frac{1}{2}\right)^{x+1}}{\left(\frac{1}{4}\right)^{x-1}}\right)^{\frac{|x+2|}{|x+1|-|x+2|}} \leq 8$,

(40) $\log_2^2 x - 3\log_4 x + \log_8 4 = \frac{1}{6}$,

(41) $\log_x 3 - 6\log_3 x \geq 1$,

(42) $|2\sin(x) - 1| - |3\sin(x) + 1| > 1$,

(43) $\mathrm{tg}^2\left(x^2 - 4x + 1\right) < 3$,

(44) $(a+1)x^2 + (a-3)x - 2a + 2 > 0$,

(45) $13\cos^2\left(\frac{x+3}{2}\right) + 8\cos\left(\frac{x+3}{2}\right) - 3\sin^2\left(\frac{x+3}{2}\right) > 0$.

Sketching a graph:

Sketch the graph of the following functions:

(46) $||x-1|^2-1|-3$,

(47) $|(\operatorname{arctg}(|x|-1)|$,

(48) $\left|(\sin\left|x+\frac{\pi}{6}\right|-\frac{1}{2}\right|$,

(49) $\left|\left(\frac{1}{2}\right)^{||x|-1|-2}-1\right|$,

(50) $\log||x|-e|$,

(51) $|2\cos(x)-1|$,

(52) $\left|\frac{x-1}{2x+1}\right|$,

(53) $|\log|x-1|-2|$,

(54) $\left|e-e^{|1-x|}\right|$,

(55) $\sin\left(|x|-\frac{\pi}{6}\right)-\frac{1}{2}$.

Results:

(1) $(4,6>$,

(2) $\left(-\infty,-10-\sqrt{97}\right\rangle\cup\left(-5,-10+\sqrt{97}\right\rangle\cup(4,+\infty)$,

(3) $\left(-3,-\frac{7}{4}\right\rangle\cup(2,+\infty)$,

(4) $\left(-3,\frac{-2-\sqrt{6}}{2}\right\rangle\cup\left\langle\frac{1}{8},2\right)$,

(5) $(-\infty,-1)\cup<0,1)\cup(5,+\infty)$,

(6) $\left\{-\frac{2}{3},2\right\}$,

(7) $\{-3,5\}$,

(8) $(-\infty,-6>\cup<-4,-2>\cup<0,+\infty)$,

(9) $<-4,12>$,

(10) $\left\{\frac{3-\sqrt{33}}{2},0\right\}$,

(11) For $a=0:-1$; $a\neq 0:\frac{-1\pm|a-1|}{a}$,

(12) For $a=0:\mathbb{R}$;
$a\neq 0:\left(-\frac{1}{a}-\frac{2}{|a|},-\frac{1}{a}-\frac{1}{|a|}\right\rangle\cup\left\langle-\frac{1}{a}+\frac{1}{|a|},-\frac{1}{a}+\frac{2}{|a|}\right)$,

(13) For $a<\frac{\sqrt{20}}{3}:\left(\frac{-3a-\sqrt{9a^2+20}}{2},+\infty\right)$; $a\geq\frac{\sqrt{20}}{3}:$
$\left(\frac{-3a-\sqrt{9a^2+20}}{2},\frac{3a-\sqrt{9a^2-20}}{2}\right)\cup\left(\frac{3a+\sqrt{9a^2-20}}{2},+\infty\right)$,

(14) $\{0\}$,

(15) $(-1,1)$,

(16) $\left\{\frac{-3\pm\sqrt{33}}{4}\right\}$,

(17) $(-\infty,1>\cup<2,+\infty)$,

(18) $(-2,2)\setminus\{0\}$,

(19) $(-\infty,-2)\cup(-1,+\infty)$,

(20) For $a\geq 3:\emptyset$; $a\in<-3,3):(-\infty,\log_2(-a+3)-2\rangle$;
$a<-3:\langle\log_2(-a-3)-2,\log_2(-a+3)-2\rangle$,

(21) $\left(-5,1-\sqrt{24}\right\rangle\cup\left\langle1+\sqrt{24},8\right)$,

(22) $\left(-3,\frac{1}{4}\left(5-\sqrt{177}\right)\right\rangle\cup\left\langle\frac{1}{4}\left(5+\sqrt{177}\right),5\right)$,

(23) $<1,2>$,

(24) $\left\{\frac{1}{2},\frac{1}{16}\right\}$,

(25) $\left\langle\frac{11}{5},3\right)\cup(4,+\infty)$,

(26) $\left\langle\frac{-3-\sqrt{33}}{2},-2\right)\cup\left(-1,\frac{-3+\sqrt{33}}{2}\right\rangle$,

(27) $(-5,-4>\cup<-2,-1)\cup(5,6>\cup<8,9)$,

(28) For $c<5:\left(-\sqrt{11-c},-\sqrt{5-c}\right\rangle\cup\left\langle\sqrt{5-c},\sqrt{11-c}\right)$;
$c\in<5,11):\left(-\sqrt{11-c},\sqrt{11-c}\right)$; $c\geq 11:\emptyset$,

(29) $\left\{k\pi,\frac{\pi}{3}+2k\pi,\frac{5\pi}{3}+2k\pi;\ k\in\mathbb{Z}\right\}$,

(30) $\left\{2k\pi,\pi-\arcsin\left(\frac{4}{5}\right)+2k\pi;\ k\in\mathbb{Z}\right\}$,

(31) $\bigcup_{k\in\mathbb{Z}}\left(\left(2k\pi+\frac{\pi}{6},2k\pi+\frac{5\pi}{6}\right)\setminus\left\{2k\pi+\frac{\pi}{2}\right\}\right)$,

(32) $\left\langle0,2-\sqrt{\frac{5}{2}}\right)\cup\left(2+\sqrt{\frac{5}{2}},4\right\rangle$,

(33) For $a>0:\mathbb{R}$; $a\leq-4:\emptyset$; $a\in(-4,0>:\bigcup_{k\in\mathbb{Z}}(2k\pi+(-\pi-\alpha,\alpha))$, where $\alpha=\arcsin\left(1-\sqrt{-a}\right)$,

(34) $\left\{2k\pi\pm\frac{2\pi}{3}+\operatorname{sign}(k);\ k\in\mathbb{Z}\setminus\{0\}\right\}\cup\left\{\pm\left(\frac{2\pi}{3}+1\right)\right\}$,

(35) $\bigcup_{k\in\mathbb{Z}}\left(2k\pi+\left(-\pi-\arcsin\left(\frac{-1+\sqrt{3}}{2}\right),\arcsin\left(\frac{-1+\sqrt{3}}{2}\right)\right)\right)$,

(36) $\left\{\frac{k\pi}{2};k\in\mathbb{Z}\right\}$,

(37) $\mathbb{R}\setminus\left\{\frac{\pi}{2}+k\pi;\ k\in\mathbb{Z}\right\}$,

(38) $\bigcup_{k\in\mathbb{Z}}\left(\frac{k\pi}{5}+\left(\frac{2-\pi}{10},\frac{\pi+3}{15}\right\rangle\right)$,

(39) $\left(-\infty,-\frac{3}{2}\right)\cup\left\langle\frac{1+\sqrt{13}}{2},+\infty\right)$,

(40) $\left\{2,\sqrt{2}\right\}$,

(41) $\left(0,\frac{1}{\sqrt{3}}\right\rangle\cup\left(1,\sqrt[3]{3}\right\rangle$,

(42) $\bigcup_{k\in\mathbb{Z}}\left(2k\pi+\left(-\pi+\arcsin\left(\frac{1}{5}\right),-\arcsin\left(\frac{1}{5}\right)\right)\setminus\left\{-\frac{\pi}{2}\right\}\right)$,

(43) Put $x_k^{\pm}=2\pm\sqrt{3+k\pi-\frac{\pi}{3}}$, $k\in\mathbb{N}_0$ and
$y_k^{\pm}=2\pm\sqrt{3+k\pi+\frac{\pi}{3}}$, $k\in\mathbb{N}_0\cup\{-1\}$. Then
$x\in\left(y_{-1}^{-},y_{-1}^{+}\right)\cup\bigcup_{k\in\mathbb{N}_0}\left(\left(y_k^{-},x_k^{-}\right)\cup\left(x_k^{+},y_k^{+}\right)\right)$,

(44) For $a<-1:\left(\frac{2-2a}{a+1},1\right)$; $a=-1:(-\infty,1)$;
$a\in\left(-1,\frac{1}{3}\right):(-\infty,1)\cup\left(\frac{2-2a}{a+1},+\infty\right)$;
$a\geq\frac{1}{3}:\left(-\infty,\frac{2-2a}{a+1}\right)\cup(1,+\infty)$,

(45) $\bigcup_{k\in\mathbb{Z}}\left(4k\pi-3+\left(-2\arccos\left(\frac{1}{4}\right),2\arccos\left(\frac{1}{4}\right)\right)\right)\cup$
$\bigcup_{k\in\mathbb{Z}}\left(4k\pi-3+\left(2\arccos\left(-\frac{3}{4}\right),4\pi-2\arccos\left(-\frac{3}{4}\right)\right)\right)$.